UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2011**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

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UNIVERSITY *of* CAMBRIDGE
International Examinations

[Turn over

1 Find the term independent of x in the expansion of $\left(2x + \frac{1}{x^2}\right)^6$. [3]

2 A curve has equation $y = 3x^3 - 6x^2 + 4x + 2$. Show that the gradient of the curve is never negative. [3]

3 **(i)** Sketch, on a single diagram, the graphs of $y = \cos 2\theta$ and $y = \frac{1}{2}$ for $0 \leqslant \theta \leqslant 2\pi$. [3]

(ii) Write down the number of roots of the equation $2\cos 2\theta - 1 = 0$ in the interval $0 \leqslant \theta \leqslant 2\pi$. [1]

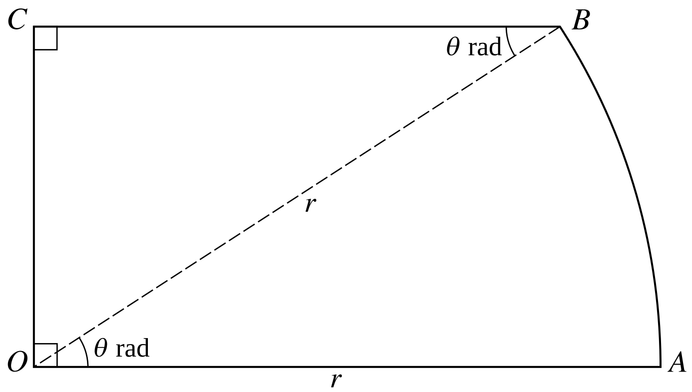
(iii) Deduce the number of roots of the equation $2\cos 2\theta - 1 = 0$ in the interval $10\pi \leqslant \theta \leqslant 20\pi$. [1]

4 A function f is defined for $x \in \mathbb{R}$ and is such that $\mathrm{f}'(x) = 2x - 6$. The range of the function is given by $\mathrm{f}(x) \geqslant -4$.

(i) State the value of x for which $\mathrm{f}(x)$ has a stationary value. [1]

(ii) Find an expression for $\mathrm{f}(x)$ in terms of x. [4]

5



The diagram represents a metal plate $OABC$, consisting of a sector OAB of a circle with centre O and radius r, together with a triangle OCB which is right-angled at C. Angle $AOB = \theta$ radians and OC is perpendicular to OA.

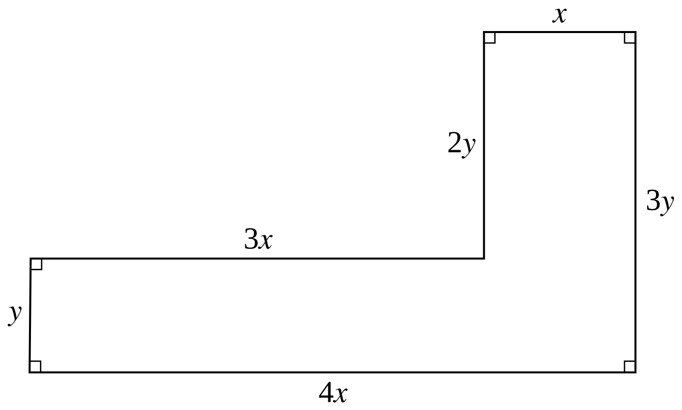
(i) Find an expression in terms of r and θ for the perimeter of the plate. [3]

(ii) For the case where $r = 10$ and $\theta = \frac{1}{5}\pi$, find the area of the plate. [3]

6 **(a)** The sixth term of an arithmetic progression is 23 and the sum of the first ten terms is 200. Find the seventh term. [4]

(b) A geometric progression has first term 1 and common ratio r. A second geometric progression has first term 4 and common ratio $\frac{1}{4}r$. The two progressions have the same sum to infinity, S. Find the values of r and S. [3]

7



The diagram shows the dimensions in metres of an L-shaped garden. The perimeter of the garden is 48 m.

(i) Find an expression for y in terms of x. [1]

(ii) Given that the area of the garden is $A\,\text{m}^2$, show that $A = 48x - 8x^2$. [2]

(iii) Given that x can vary, find the maximum area of the garden, showing that this is a maximum value rather than a minimum value. [4]

8 Relative to an origin O, the point A has position vector $4\mathbf{i} + 7\mathbf{j} - p\mathbf{k}$ and the point B has position vector $8\mathbf{i} - \mathbf{j} - p\mathbf{k}$, where p is a constant.

(i) Find $\overrightarrow{OA} \cdot \overrightarrow{OB}$. [2]

(ii) Hence show that there are no real values of p for which OA and OB are perpendicular to each other. [1]

(iii) Find the values of p for which angle $AOB = 60°$. [4]

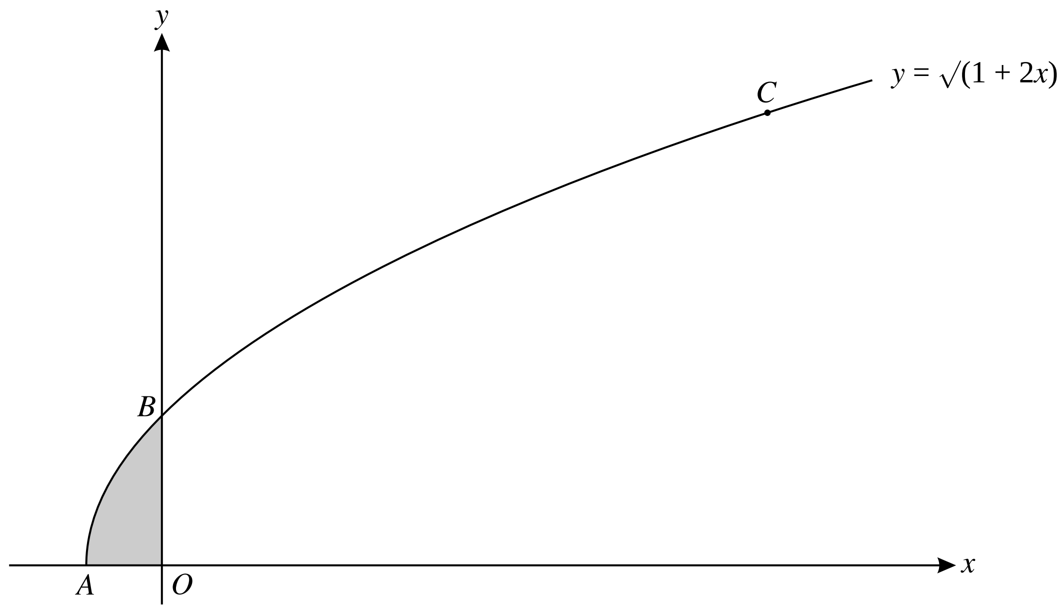
9 A line has equation $y = kx + 6$ and a curve has equation $y = x^2 + 3x + 2k$, where k is a constant.

(i) For the case where $k = 2$, the line and the curve intersect at points A and B. Find the distance AB and the coordinates of the mid-point of AB. [5]

(ii) Find the two values of k for which the line is a tangent to the curve. [4]

[Questions 10 and 11 are printed on the next page.]

10



The diagram shows the curve $y = \sqrt{(1 + 2x)}$ meeting the x-axis at A and the y-axis at B. The y-coordinate of the point C on the curve is 3.

(i) Find the coordinates of B and C. [2]

(ii) Find the equation of the normal to the curve at C. [4]

(iii) Find the volume obtained when the shaded region is rotated through $360°$ about the ***y*-axis**. [5]

11 Functions f and g are defined by

$$\mathrm{f} : x \mapsto 2x^2 - 8x + 10 \quad \text{for } 0 \leqslant x \leqslant 2,$$
$$\mathrm{g} : x \mapsto x \quad \text{for } 0 \leqslant x \leqslant 10.$$

(i) Express $\mathrm{f}(x)$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

(ii) State the range of f. [1]

(iii) State the domain of f^{-1}. [1]

(iv) Sketch on the same diagram the graphs of $y = \mathrm{f}(x)$, $y = \mathrm{g}(x)$ and $y = \mathrm{f}^{-1}(x)$, making clear the relationship between the graphs. [4]

(v) Find an expression for $\mathrm{f}^{-1}(x)$. [3]